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Parametric Stability Analysis of a Thin Shell Structure Under Own Weight

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***Abstract:** In this study, the internal forces, parametric stability and free vibration of elliptic parabolic shell analysis are performed by theoretical and finite element methods. Geometrical and material linearity is considered in modeling and analysis of the structure. Theoretical analysis is done by shallow shell theory and the finite element analysis is carried out by using DLUBAL RFEM 5.3.1. Theoretical formulas in this study are coded by the MATLAB program. The parameter considered thickness, dimension, radius of curvature, modulus of elasticity and side height of the shell for determining the critical buckling load and thickness for free vibration. The internal forces, displacement, critical buckling load and free vibration analysis of result in the study presented graphically and discussed in detail. The result of the finite element analysis is then compared with the theoretical analysis results.*

***Keywords:** Elliptic Parabolic Shell, Parametric Stability, Free Vibration Analysis, Theoretical Analysis, DLUBAL RFEM Analysis, MATLAB program.*

I. INTRODUCTION

Reinforced concrete thin shells can be defined as curved slabs whose thicknesses are small compared to their other dimensions like the radius of curvature. Due to its initial curvature, a shell can transfer an applied load by in-plane as well as out-of-plane actions. A thin shell subjected to an applied load, therefore, produces mainly in-plane actions, which are called membrane forces. These membrane forces are resultants of normal stresses and in-plane shear stresses that are uniformly distributed across the thickness. Deformable bodies may become unstable under certain loading conditions and thus have a premature failure. The phenomenon of instability is particularly important for thin shells subjected to compressive forces. Critical buckling load and free vibration of cylindrical and some other shells were studied extensively by many researchers, very less work was carried out on the buckling and free vibration characteristics of doubly curved shells. The study of buckling and free vibration behavior of doubly curved elliptic paraboloid, hyperbolic paraboloid, conidial and hypar shells is yet to be carried out. Generally, to determine the stability of shells is the main concern for the design of the reinforced concrete shell structures now a day.

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The present work is, therefore, expected to investigate the critical buckling and the free vibration behavior of the elliptic paraboloid shell by employing the theoretical and finite element method. The finite element software RFEM is used for modeling and analysis of thin elliptic paraboloid shell [1]. For theoretical analysis of the thin shell MATLAB R2019b [2] is essential in this research.

II. MATERIAL

For this study C20/25, C40/45 and C60/75 concrete types [3] are taken for the determination of critical buckling load. For the analysis of internal forces and free vibration structural concrete C20/25 with a young's modulus of 30 GPa, poisson's ratio of 0.2 and a unit weight of 25 kN/m^3 were used. The partial safety factors for all the reinforced concrete resistance were taken as γm=1.

III. GEOMETRY

The geometrical property and the procedures followed in the theoretical and numerical analysis of each modeled shell is described in a sample solved problem. T. Nagy [4] considered geometric details of the elliptic curve generator, which is a Positive Gaussian curvature.

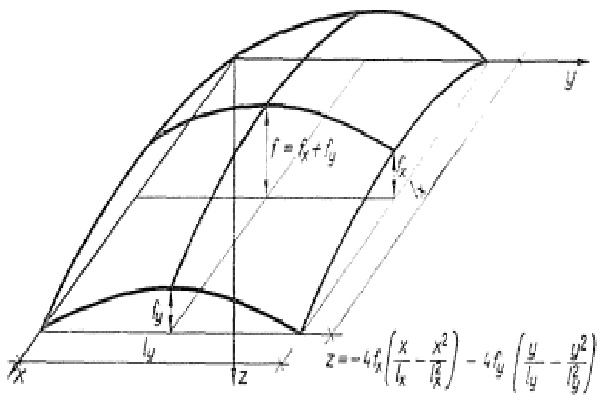

Figure 1. Elliptic paraboloid shell geometry (T. Nagy 1976)

The general equation for elliptic paraboloid shell according to T. Nagay

$$z = 4f_x\left(\frac{x}{l_x} - \frac{x^2}{(lx)^2}\right) - 4f_y\left(\frac{y}{l_y} - \frac{y^2}{(ly)^2}\right) \quad \text{.....................1}$$

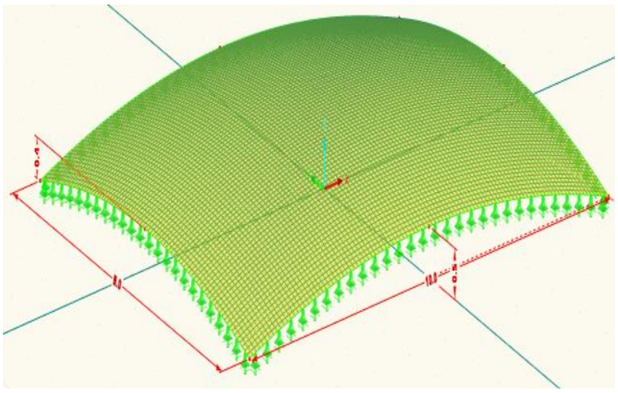

Figure 2. Elliptic paraboloid shell model by RFEM

IV. THEORY OF SHALLOW SHELLS

A shallow shell is defined as a shell having a relatively small raise as compared to its spans. A shell is said to be shallow if at any point of its middle surface the following inequalities hold:

$$\left(\frac{dz}{dx}\right)^2 \approx 1, \qquad \left(\frac{dz}{dy}\right)^2 \approx 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 2$$

where $z = z\,(x, y)$ represents the equation of the shell middle surface.

The theory of shallow shells can be also used to analyze shells that become locally shallow when the original shell is divided into finite segments or elements [5]. If confine the analysis to the accuracy of the theory of thin shells, i.e., consider a shell as shallow for

$$\left(\frac{dz}{dx}\right)^2 \prec 0.05 \left(\frac{dz}{dy}\right)^2 \prec 0.05 \quad \ldots\ldots\ldots\ldots 3$$

The DMV (Donnell–Mushtari–Vlasov) theory can be useful to the analysis of generally shallow shells form. Thus, the system of the governing differential equations of the estimated DMV theory of thin shells have the following form:

$$\begin{aligned} &D\nabla^2\nabla^2\omega - \Phi\nabla_k^2 = P_3 \\ &\nabla^2\nabla^2\Phi - Eh\nabla_k^2 = 0 \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

Determined the internal forces and moments by shallow shell theory.

$$\begin{aligned} &N_1 = \frac{\partial^2\Phi}{\partial y^2},\; N_2 = \frac{\partial^2\Phi}{\partial x^2},\; N_{12} = \frac{\partial^2\Phi}{\partial x\partial y} \\ &M_1 = -D\left(\frac{\partial^2\omega}{\partial x^2} + \nu\frac{\partial^2\omega}{\partial y^2}\right),\; M_2 = -D\left(\frac{\partial^2\omega}{\partial y^2} + \nu\frac{\partial^2\omega}{\partial x^2}\right) \end{aligned} \quad \ldots\ldots 5$$

V. STABILITY ANALYSIS

The stability of any deformable bodies that may become unstable under certain loading conditions and thus have a premature failure. The phenomenon of instability is particularly important for thin shells subjected to compressive forces. The design of thin shells is normally dominated by the stability considerations and not merely the material strength requirements. Hence, the stability analysis of thin shells acquires prime importance in various problems related to the design of shells [6].

The theoretical buckling load for a doubly curved elastic shell under the dead load, is

$$p_{cr} = \frac{2Et^2}{\sqrt{3\left(1-\nu^2\right)}}\frac{1}{R_1}\frac{1}{R_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

used in this research paper for the determination of critical pressure p_{cr} for an elliptical paraboloid shell.

VI. FREE VIBRATION

The governing differential equations of free vibrations of shallow shells.

$$\begin{aligned} &D\nabla^2\nabla^2 w - \nabla_k^2\Phi + \rho h\frac{\partial^2 w}{\partial t^2} = 0 \\ &\nabla^2\nabla^2\Phi - Eh\nabla_k^2 w = 0 \end{aligned} \quad \ldots\ldots\ldots\ldots\ldots 7$$

The natural frequency of free vibrations of a shallow shell for the simply supported shell of double curvature.

$$\omega_{mn}^2 = \frac{1}{\rho h}\left[D\left(\lambda_{mn}^2 + \mu_{mn}^2\right)^2 + \frac{Eh\left(\frac{\lambda_n^2}{R_1} + \frac{\mu_m^2}{R_2}\right)^2}{\left(\lambda_n^2 + \mu_m^2\right)^2}\right] \quad \ldots\ldots 8$$

Free vibration of simply supported doubly curved shell is analyzed by [7]

$$\omega_{mnc}^2 = \omega_{mnf}^2 + \frac{\left[\frac{1}{R_y}\left(\frac{m\pi}{a}\right)^2 + \frac{1}{R_x}\left(\frac{n\pi}{b}\right)^2\right]^2}{\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^2}\left(\frac{E}{\rho}\right) \quad \ldots\ldots 9$$

where ω_{mnf}^2 is the equivalent frequency for the flat plate

$$\omega_{mnf} = \pi^2\left[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2\right]\sqrt{\frac{D}{\rho h}} \quad \ldots\ldots\ldots\ldots 10$$

VII. PARAMETRIC STUDY

The impact of thickness, the radius of curvature, rise of curvature in x and y direction and the dimension of elliptic parabolic in terms of the square and rectangular plan in critical buckling of the load. and also the influence of thickness on natural vibration of the shell by its weight. For the analysis of buckling load and natural vibration simply supported boundary condition is considered.

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Table 1. Geometrical Parameter for Models

Description	Symbols	Values(m)
The thickness of the shell	h	0.050 - 0.150
Rise of the shell in the x-direction	f_1 or f_x	0.500 - 0.800
Rise of the shell in the y-direction	f_2 or f_y	0.500
The dimension of the shell in the x-direction	a	5.000
The dimension of the shell in the y-direction	b	4.000 and 5.000
The radius of curvature in rectangular plan Elliptic paraboloid shell x-direction	R_1 or R_x	15.625 and 17.857
The radius of curvature in rectangular plan Elliptic paraboloid shell y-direction	R_2 or R_y	16.000
The radius of curvature in square plan elliptic Paraboloid shell x-direction	R_1 or R_x	15.625 - 25.000
The radius of curvature in square plan elliptic Paraboloid shell y-direction	R_2 or R_y	15.625 - 25.000

VIII. ANALYSIS RESULT

Table 2. Theoretical and RFEM result (a=5m, b=5m, f_1=0.5m, f_2=0.5m for C20/25)

					Computing by table		Computing by RFEM	
x/a	***y/b***	***x (m)***	***y* (m)**	***K***	***Nx* (kN/m)**	***Ny* (kN/m)**	***Nx* (kN/m)**	***Ny* (kN/m)**
1.000	0.000	-5.000	0.000	1.020	0.000	-61.286	-0.293	-0.059
0.750	0.000	-3.750	0.000	1.011	-12.766	-49.323	-13.518	-38.404
0.500	0.000	-2.500	0.000	1.005	-22.863	-39.553	-23.970	-43.640
0.250	0.000	-1.250	0.000	1.001	-29.161	-33.333	-30.349	-36.382
0.000	0.000	0.000	0.000	1.000	-31.250	-31.250	-32.519	-32.520
0.250	0.000	1.250	0.000	1.001	-29.161	-33.333	-30.566	-36.023
0.500	0.000	2.500	0.000	1.005	-22.863	-39.553	-24.418	-43.370
0.750	0.000	3.750	0.000	1.011	-12.766	-49.323	-14.161	-39.412
1.000	0.000	5.000	0.000	1.020	0.000	-61.286	-0.293	-0.059
0.000	1.000	0.000	-5.000	0.981	-61.286	0.000	-0.059	-0.293
0.000	0.750	0.000	-3.750	0.989	-49.323	-12.766	-38.404	-13.518
0.000	0.500	0.000	-2.500	0.995	-39.553	-22.863	-43.640	-23.970
0.000	0.250	0.000	-1.250	0.999	-33.333	-29.161	-36.382	-30.349
0.000	0.000	0.000	0.000	1.000	-31.250	-31.250	-32.520	-32.519
0.000	0.250	0.000	1.250	0.999	-33.333	-29.161	-36.023	-30.566
0.000	0.500	0.000	2.500	0.995	-39.553	-22.863	-43.370	-24.418
0.000	0.750	0.000	3.750	0.989	-49.323	-12.766	-39.412	-14.161
0.000	1.000	0.000	5.000	0.981	-61.286	0.000	-0.059	-0.293

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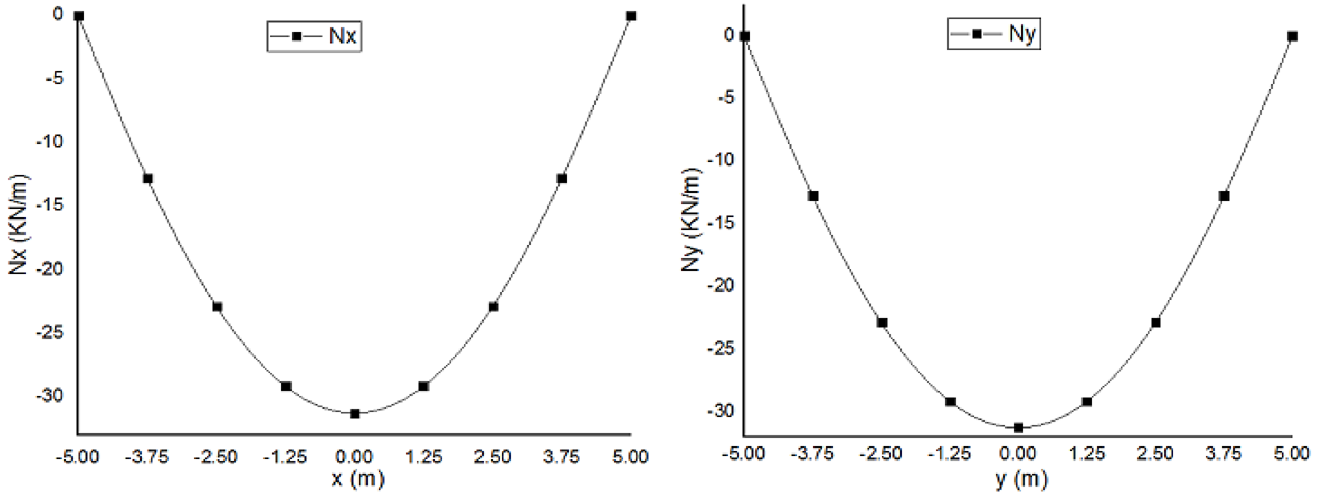

Figure 3. Internal Force Diagram in The Central Section of the Shell

Table 3. Internal Force and Moment Analysis Result (*a*=5m, *b*=5m, f_1=0.5m, f_2=0.5m for C20/25)

		Computing by (DMV) theory				Computing by RFEM			
x	*y*	*Nx*	*Ny*	*Mx*	*My*	*Nx*	*Ny*	*Mx*	*My*
0.000	5.000	0.000	0.000	0.000	0.000	-0.293	-0.059	0.000	0.012
1.250	5.000	-13.346	-38.360	0.565	0.096	-13.518	-38.404	0.573	0.085
2.500	5.000	-23.947	-43.394	0.204	0.008	-23.970	-43.640	0.215	0.003
3.750	5.000	-30.421	-36.205	-0.012	-0.046	-30.349	-36.382	-0.009	-0.047
5.000	5.000	-32.558	-32.558	-0.059	-0.059	-32.519	-32.520	-0.059	-0.059
6.250	5.000	-30.421	-36.205	-0.012	-0.046	-30.566	-36.023	-0.014	-0.049
7.500	5.000	-23.947	-43.394	0.204	0.008	-24.418	-43.370	0.197	-0.001
8.750	5.000	-13.346	-38.360	0.565	0.096	-14.161	-39.412	0.568	0.082
10.000	5.000	0.000	0.000	0.000	0.000	-0.293	-0.059	0.000	0.012
5.000	0.000	0.000	0.000	0.000	0.000	-0.059	-0.293	0.012	0.000
5.000	1.250	-38.360	-13.346	0.096	0.565	-38.404	-13.518	0.085	0.573
5.000	2.500	-43.394	-23.947	0.008	0.204	-43.640	-23.970	0.003	0.215
5.000	3.750	-36.205	-30.421	-0.046	-0.012	-36.382	-30.349	-0.047	-0.009
5.000	5.000	-32.558	-32.558	-0.059	-0.059	-32.520	-32.519	-0.059	-0.059
5.000	6.250	-36.205	-30.421	-0.046	-0.012	-36.023	-30.566	-0.049	-0.014
5.000	7.500	-43.394	-23.947	0.008	0.204	-43.370	-24.418	-0.001	0.197
5.000	8.750	-38.360	-13.346	0.096	0.565	-39.412	-14.161	0.082	0.568
5.000	10.000	0.000	0.000	0.000	0.000	-0.059	-0.293	0.012	0.000

Table 4. Deformation in the central section of a square elliptic parabolic shell

	DMV	RFEM
x	*u* (mm)	*u* (mm)
-5.000	0.000	0.072
-3.750	0.431	0.439
-2.500	0.561	0.577
-1.250	0.555	0.574
0.000	0.543	0.561
1.250	0.555	0.574
2.500	0.561	0.577
3.750	0.431	0.439
5.000	0.000	0.072

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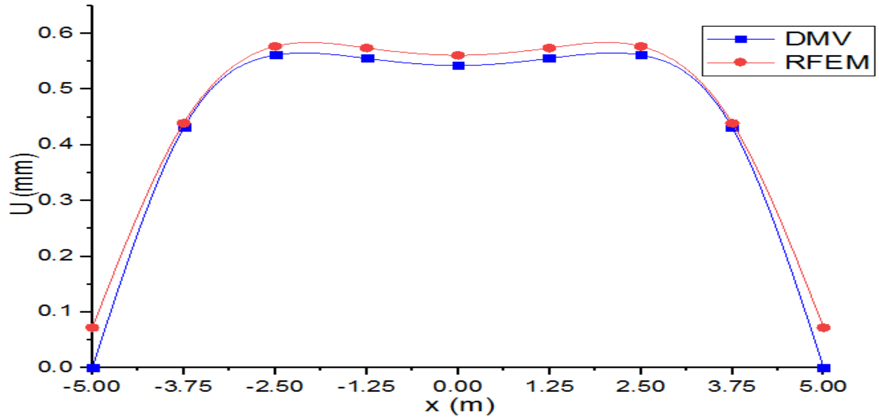

Figure 4. Deformation in the Central Section x and y-Direction

IX. CRITICAL BUCKLING LOAD

The percentile variation of the value of the critical buckling load of the shell that is done by theoretically and numerically is increased when the shallowness of the shell is increased in square plan elliptic parabolic shell with the thickness of the shell is (5 – 15cm) this is seen in the graph. The term shallowness means the side raise of the elliptic paraboloid shells.

The relation between buckling load and radius of curvature of the shell is inversely this is clearly shown in the result that gained by both analysis methods. When the radius of curvature decreased from 25m to 15.625m in the opposite way the critical buckling load increased 139.320 to 334.245 kN/m^2.

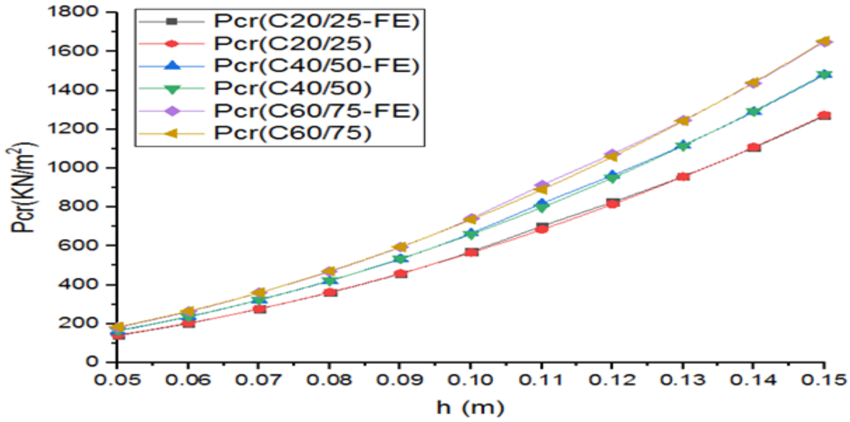

Figure 5 Critical Buckling Load by RFEM & DMV

Table 5. Critical Load with Parameters (*a*=*b*=5.0m, *f*$_1$=*f*$_2$=0.5m and *R*$_1$=*R*$_2$=25.0m)

Concrete Type	C20/25		C40/50		C60/75		
Thickness (m)	**Pcr RFEM**	**Pcr (kN/m^2)**	**Pcr RFEM**	**Pcr (kN/m^2)**	**Pcr RFEM**	**Pcr (kN/m^2)**	**Difference in %**
0.050	139.320	141.421	162.540	164.992	181.116	183.848	-1.486
0.060	201.938	203.647	235.594	237.588	262.519	264.741	-0.839
0.070	276.426	277.186	322.497	323.384	359.354	360.342	-0.274
0.080	360.722	362.039	420.842	422.378	468.939	470.650	-0.364
0.090	457.000	458.205	533.167	534.573	594.100	595.667	-0.263
0.100	570.418	565.685	665.488	659.966	741.543	735.391	0.837
0.110	701.689	684.479	818.637	798.559	912.196	889.823	2.514
0.120	825.676	814.587	963.289	950.352	1073.379	1058.963	1.361
0.130	958.761	956.008	1118.555	1115.343	1246.389	1242.811	0.288
0.140	1105.987	1108.743	1290.318	1293.534	1437.783	1441.366	-0.249
0.150	1268.845	1272.792	1480.319	1484.924	1649.499	1654.630	-0.310

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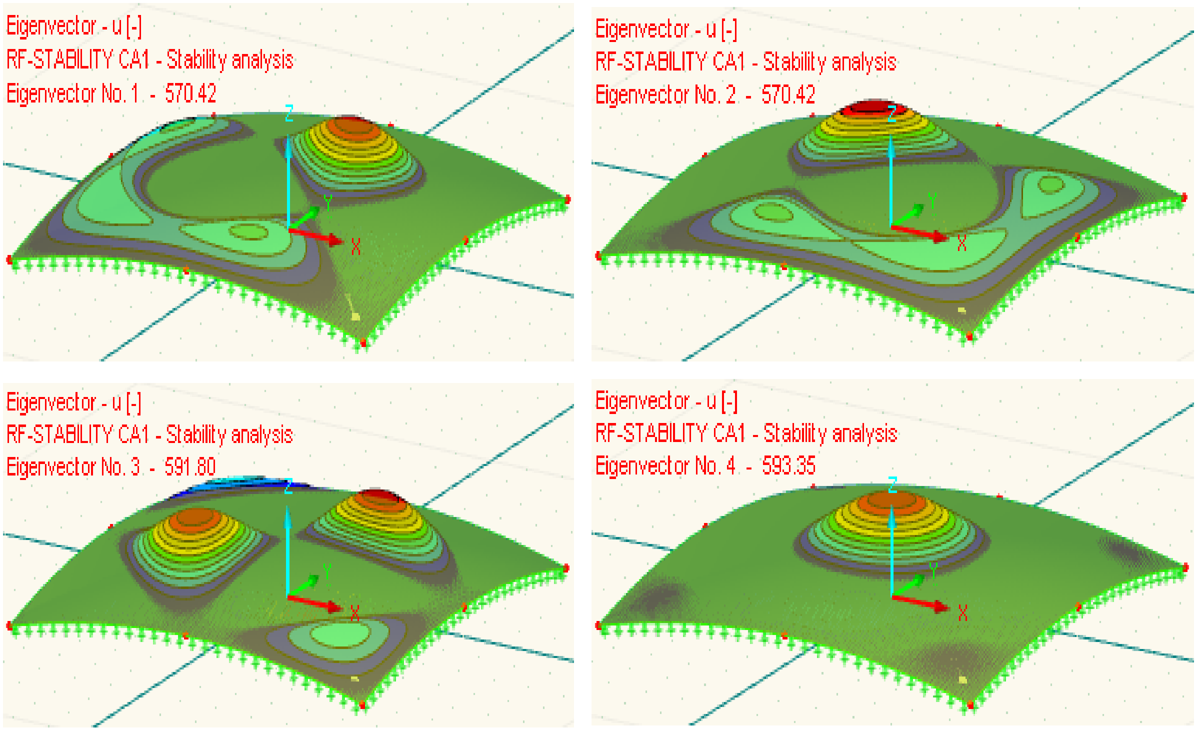

Figure 6. Mode shape with corresponding critical load (*a*=*b*=5.0m, f_1=f_2=0.5m, *t*=0.1m and R_1=R_2=25.0m)

In general, the shallowness, the radius of curvature, type of concrete and thickness of the shell has a significant impact on the critical buckling load of the shell.

X. FREE VIBRATION ANALYSIS

The natural vibration cases tab is the centerpiece of the RF-DYNAM Pro - natural vibrations module. It is essential for the response spectrum analysis and the time history analysis based on modal decomposition (modal analysis) [8].

The equation of motion of a multi-degree of freedom without damping is solved with the four available eigenvalue solvers. The equation of motion is defined as

$$M\ddot{u} + Ku = 0$$

where *M* is the mass matrix, *K* is the stiffness matrix and *u* are he mode shapes containing translational and rotational parts:

$$u = \left(u_x, u_y, u_z, \varphi_x, \varphi_y, \varphi_z\right)^T$$

The eigenvalue λ [1/s2] is connected to the angular frequency ω [1/s] with $\lambda i = \omega i^2$. The natural frequency *f* [Hz] is then derived with $f = \omega/2\pi$, and the natural period *t* [s] is the reciprocal of the frequency obtained with $t = 1/f$.

Table 6. Mode result by theoretical and RFEM (*a*=*b*=5.0m, f_1=f_2=0.5m and R_1=R_2=25.0m)

C20/25							
Thick ness(m)	*D* (N.m)	Theoretical result			RFEM result		
		ω (rad/s)	*f* (Hz)	*t* (s)	*ω* (rad/s)	*f* (Hz)	*t* (s)
0.050	325520.833	138.930	22.111	0.045	145.812	23.207	0.043
0.060	562500.000	139.090	22.137	0.045	148.886	23.696	0.042
0.070	893229.167	139.280	22.167	0.045	152.302	24.240	0.041
0.080	1333333.333	139.498	22.202	0.045	154.954	24.662	0.041
0.090	1898437.500	139.745	22.241	0.045	156.733	24.945	0.040
0.100	2604166.667	140.021	22.285	0.045	158.522	25.230	0.040
0.110	3466145.833	140.325	22.333	0.045	160.357	25.522	0.039
0.120	4500000.000	140.657	22.386	0.045	162.260	25.824	0.039
0.130	5721354.167	141.017	22.444	0.045	164.244	26.140	0.038
0.140	7145833.333	141.405	22.505	0.044	166.316	26.470	0.038
0.150	8789062.500	141.821	22.572	0.044	168.480	26.814	0.037

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XI. CONCLUSION

Modeling and analyzing of the shell by RFEM finite element software that helps me to determine the internal forces, critical buckling loads and free vibration. The results gained from this software also good resemble the theoretical results except for the free vibration. Results demonstrate that the theoretical result does not necessarily give the same solution to the finite element analysis. It has been speculated that this might be due to assumptions during drive a theoretical formula. From this, it has decided that this study required experimental work.

DECLARATION

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Conflicts of Interest/ Competing Interests	No conflicts of interest to the best of our knowledge.
Ethical Approval and Consent to Participate	No, the article does not require ethical approval and consent to participate with evidence.
Availability of Data and Material/ Data Access Statement	Not relevant.
Authors Contributions	I am only the sole author of the article.

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